THE 8th INTERNATIONAL CONFERENCE ON COMPRESSORS AND REFRIGERATION, 2017

Sponsored By Xi' an Jiaotong University, **July 20-22, 2017**

International Institute of Refrigeration Conference Series

RAY W. HERRICK LABORATORIES

PDSim: A Generalized Modeling Platform to Predict the Performance of Positive Displacement Compressors and Expanders

Davide Ziviani, Eckhard A. Groll

Purdue University,

Ray W. Herrick Laboratories,

West Lafayette, Indiana 47907, USA

E-mail: dziviani@purdue.edu, groll@purdue.edu

Outline

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Outline

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Introduction (1/3)

Ray W. Herrick Laboratories

Positive displacement compressor (and expander) modeling provides several benefits:

- Fast, inexpensive method for evaluating the performance of different compressor designs
- Identifies the source of losses, enabling design modifications to improve performance
- Can be used to generate compressor maps for use in system modeling

Experimental testing is still required to verify the model predictions under certain operating conditions

- The model can be used to investigate broad range of operating conditions
- The model can be tuned to better describe the real behavior of the compressor

Introduction (2/3)

A positive displacement compressor model follows a general structure that can be divided in two parts:

- Generalized approach
  - Governing equations applied to a (or multiple) control volume(s)
  - Integration scheme of governing equations
  - Thermodynamic and transport properties library
  - Overall energy balance
- Compressor-type specific elements
  - Geometry (volume curve, chamber wall area, sealing line length, etc.)
  - Flow models (suction and discharge processes, valves, leakage flows, etc.)
  - Heat transfer mechanisms (heat transfer within the chamber, with the compressor shell, with the environment, etc.)
  - Friction losses (mechanical analysis of bearings, sliding contacts, rolling contacts, lubrication effects, etc.)

Introduction (3/3)

RAY W. HERRICK LABORATORIES

In addition, there are other aspects that can be accounted for:

- Hermetic, semi-hermetic or open-drive designs
- Gas pulsations (sound and vibration analyses)
- Interaction with secondary fluid within the compressor shell (e.g. lubricant oil)
- Compressor performance enhancements (e.g., vapor-injection, oil-injection, etc.)
- Two-phase flow conditions (e.g., wet expansion, slugging phenomena, etc.)

Outline

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

History of Compressor Modeling at the Ray W. Herrick Laboratories (1/2)

- Fundamentals of computer simulation of positive displacement compressors were described by Prof. Soedel and Prof. Hamilton in their short-course at the Purdue Conferences (1972 and 1974)
- Such approach was applied systematically to different compressor types to develop tailor-made simulations models
- Prof. Groll greatly contributed to the development of compressor models for more than twenty years
- With the knowledge gained by modeling different types of compressors and expanders, it was possible to identify a general structure in the construction of their models

History of Compressor Modeling at the Ray W. Herrick Laboratories (2/2)

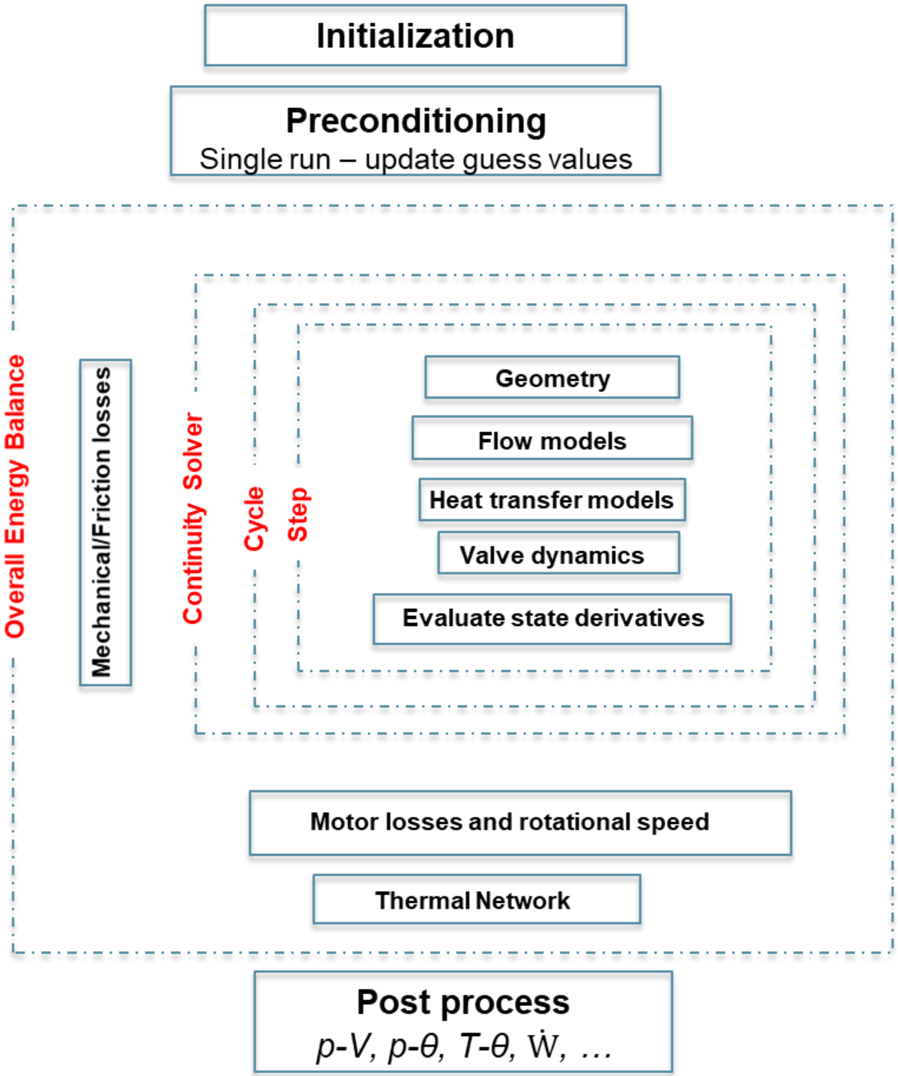
Compressor Type	PhD Theses	MSME Theses	Journal Publications	Modeling	Experiments
Scroll	3 (Chen 2000, Bell 2011, Shaffer 2012)	2 (Ramaraj 2012, Song 2013)	7	✓	✓
Reciprocating	1 (Bilal 2011)	1 (Hubacher 2003)	2	✓	✓
Linear compressor	1 (Bradshaw 2012)		2	✓	✓
Z-compressor	1 (Jovane 2007)		1 (Conf.)	✓	✓
Bowtie	1 (Kim 2005)		1	✓	(with recip.)
Miniature-scale Diaphragm	1 (Sathe 2008)		3	✓	(with half prototype)
Rolling piston	1 (Mathison 2008)		2	✓	✓
Rotary Spool	2 (Mathison 2008, Khrishna 2015)		3	✓	✓
S-RAM	1 (Yang 2017)		3 (Conf.)	✓	✓
Screw	1 (Ziviani 2017)	1 (Bein 1980)		✓	

Outline

RAY W. HERRICK LABORATORIES

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

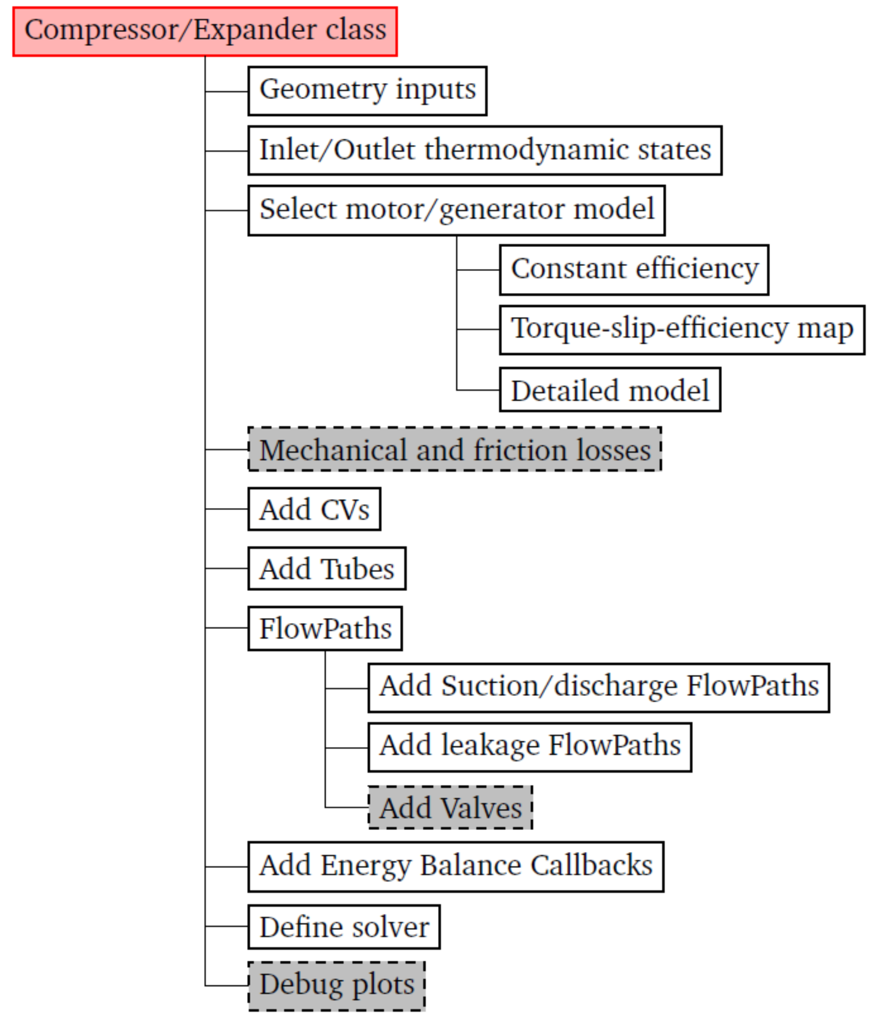
Generalized Modeling Structure (1/2)



Generalized Modeling Structure (2/2)

Desired elements on a modeling point of view:

- General structure that can be readily applied to any PD machine.
- Versatile coding scheme for quick implementation with templates available
- Library of core elements of a PD model, e.g., tubes, flow paths, CVs.
- Library of core models, e.g., valves, heat transfer, leakage, mechanical elements
- Robust solution scheme
- Facilitate the hard coding learning process

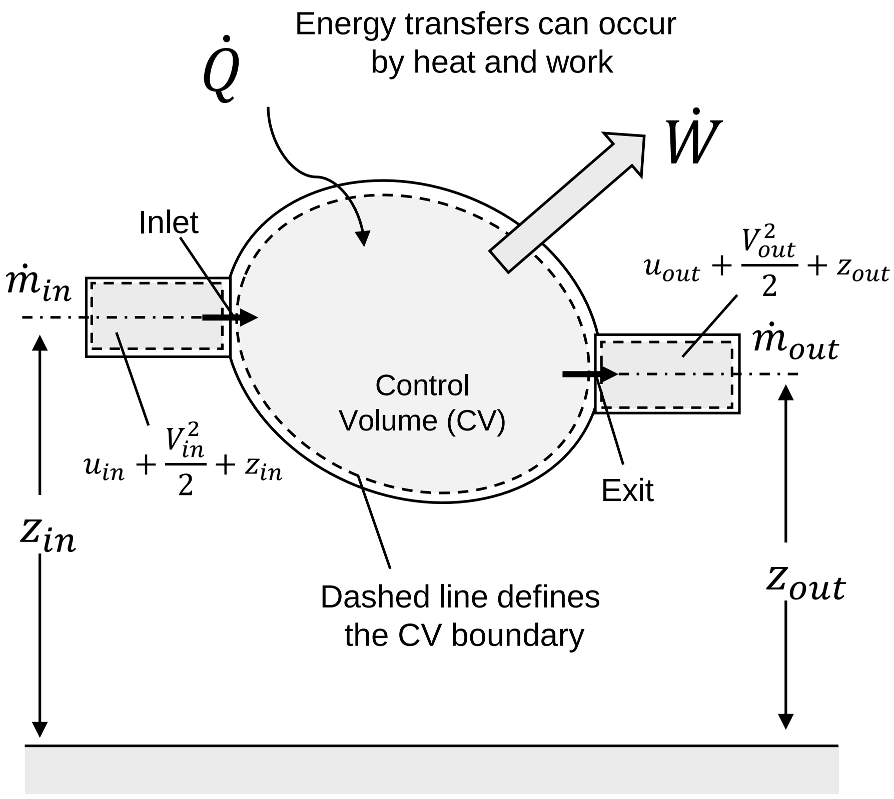


Outline

RAY W. HERRICK LABORATORIES

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Control Volume Analysis (1/5)



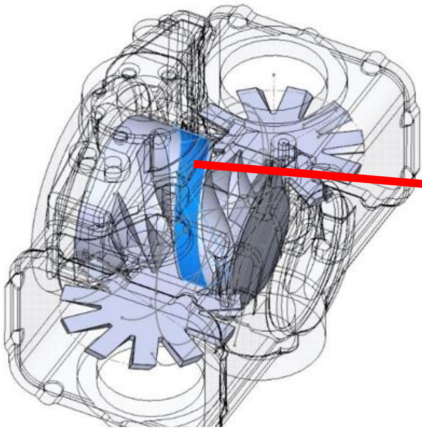
Adapted from: Morgan and M.J., Shapiro, H.N., "**Fundamentals of Engineering Thermodynamics**", *5th Edition*. Wiley, 2006

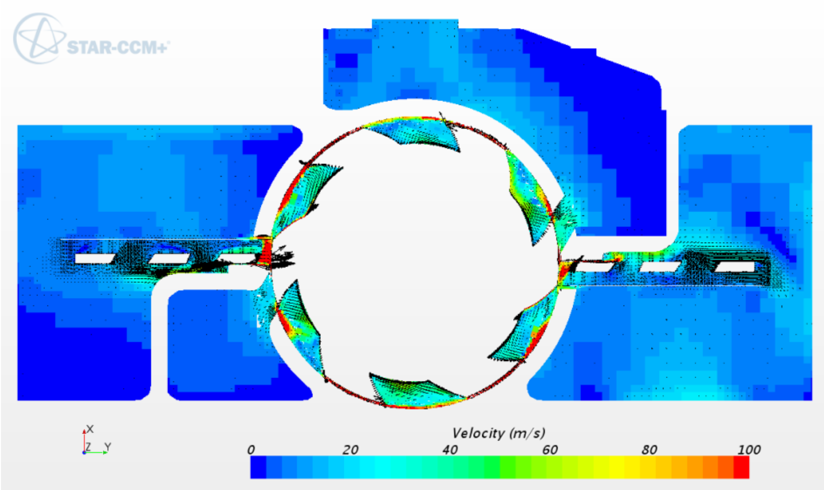
Assumptions:

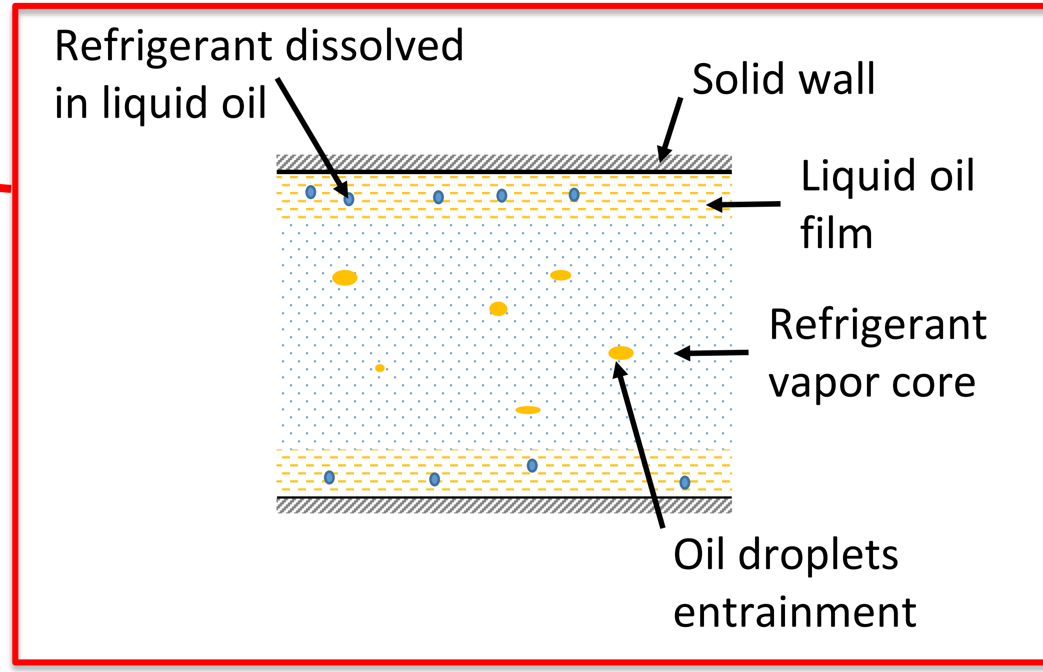
- Uniform flow
- Instantaneous mixing
- Quasi-equilibrium process (uniform pressure acting on boundary)
- Negligible changes in kinetic and potential energy

Control Volume Analysis (2/5)

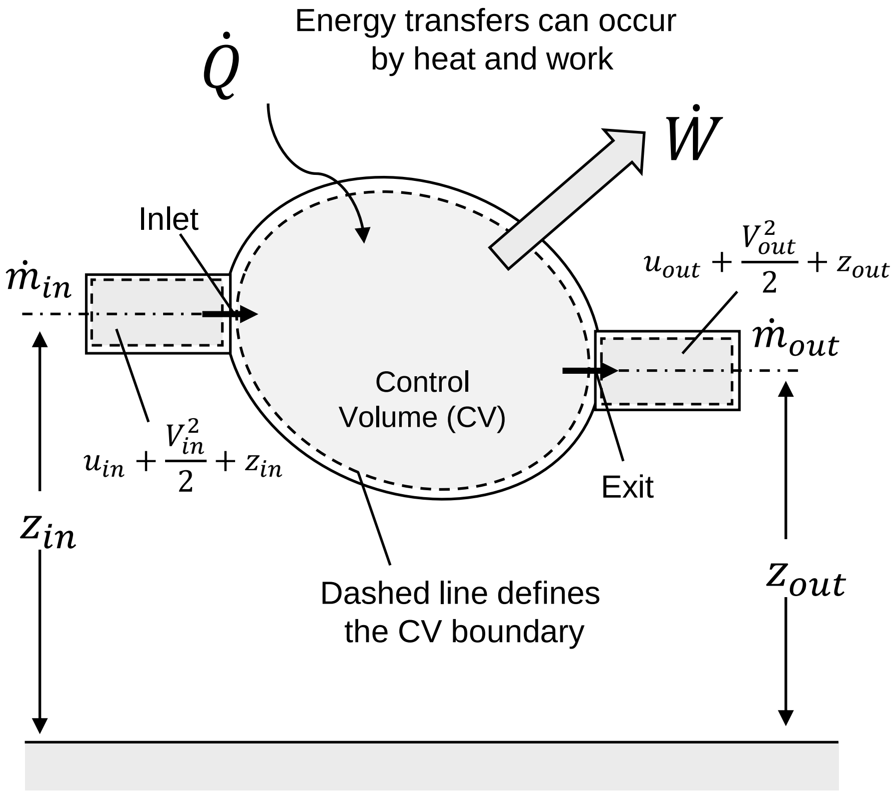
- In reality, the CV of a positive displacement is more complicated:







Control Volume Analysis (3/5)



Adapted from: Morgan and M.J., Shapiro, H.N., "**Fundamentals of Engineering Thermodynamics**", *5th Edition*. Wiley, 2006

- General form of mass balance:

$$\frac{dm_{CV}}{dt} = \sum_i \dot{m}_{in} - \sum_i \dot{m}_{out}$$

- General form of energy balance:

$$\frac{dE_{CV}}{dt} = \sum_i \dot{m}_{in} h_{in} - \sum_i \dot{m}_{out} h_{out} + \dot{Q} - \dot{W}$$

Control Volume Analysis (4/5)

Core structure of PDSim is based on two modelling approaches

- Positive displacement machines with a crank-angle motion
  - One complete working cycle and steady-periodic dynamic solution is considered
  - Governing equations are expressed in terms of crank-angle

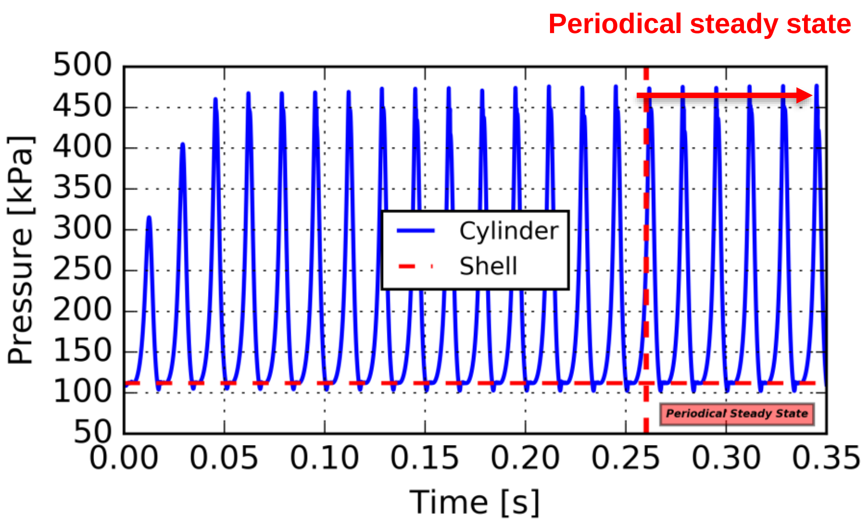
$$\mathrm{d}\theta = \omega dt$$

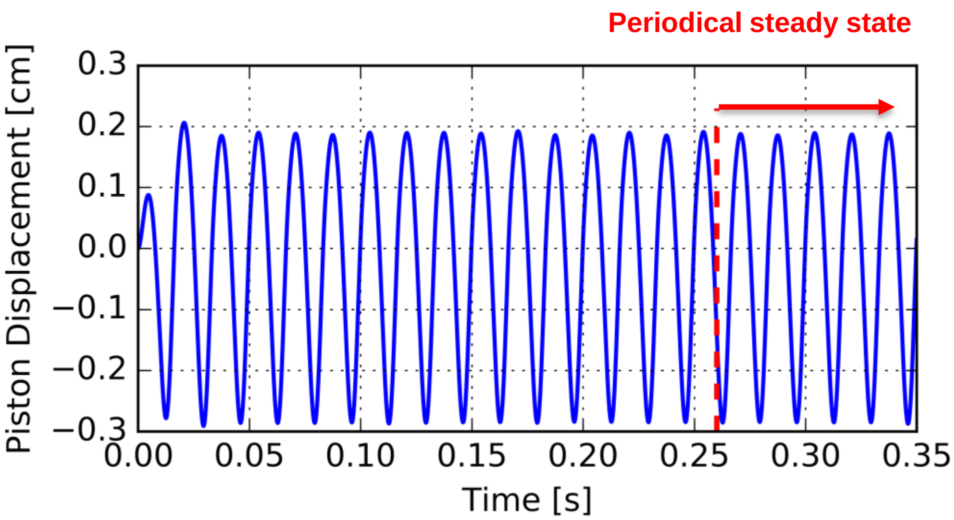
- Positive displacement machines based on linear motion (linear compressor)
  - Simulation of linear compressors requires the analysis of the dynamic process from the initial condition t=0 until the steady-periodic condition is reached
  - Transient start-up performance needs to be predicted and working cycle is established dynamically over time

Control Volume Analysis (5/5)

RAY W. HERRICK LABORATORIES

- For positive displacement compressors performing a linear motion (linear compressor), a criterion had to be developed to identify the steady-periodic solution:



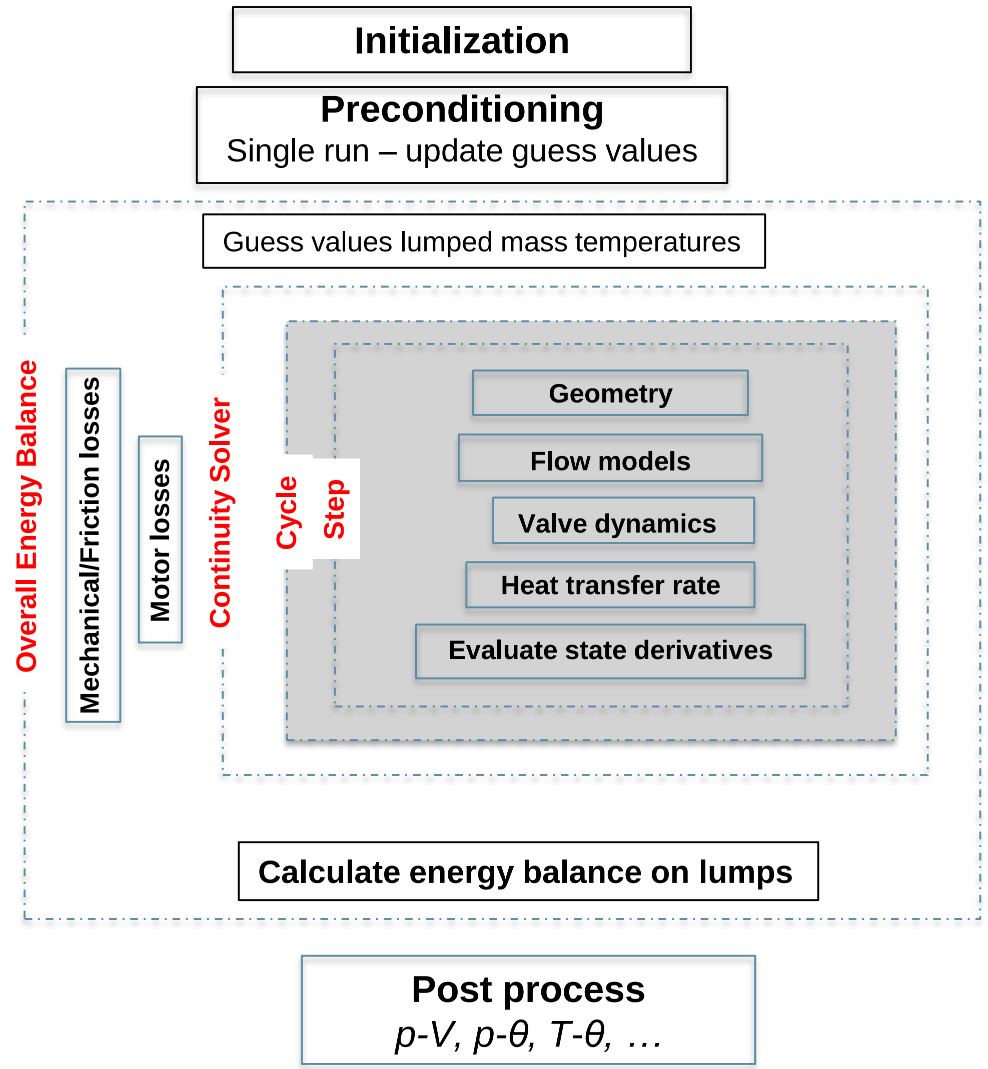


Outline

RAY W. HERRICK LABORATORIES

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Steady-periodic modeling



Thermophysical Properties (1/3)

Thermodynamic and transport properties are needed to evaluate the working process of a compressor and its performance

Thermodynamic properties are characterized by an equation of state (EOS)

Transport properties are usually described by empirical formulations or by predictive schemes such as extended corresponding states (ECS) when little or no experimental data is available.

Common independent variables to be used in EOS:

- Pressure and temperature (easy to measure)
- Temperature and density

In many applications, other variables are more appropriate:

- Cycle analysis and dynamic cycle analysis: (p,h)
- Mechanistic model of positive-displacement compressors: (T,u)

Thermophysical Properties (2/3)

RAY W. HERRICK LABORATORIES

Common used libraries for compressor modeling:

- REFPROP (Lemmon et al., 2013) developed at the National Institute of Standards and Technology (NIST)
  - It represents the state-of-the-art library for the thermophysical properties of pure fluids and mixtures
  - Available for many programming languages and environments

- CoolProp library (Bell et al., 2014) is a open-source library of thermophysical properties that emulates much of the functionality of REFPROP

Thermophysical Properties (3/3)

Two independent properties (e.g. temperature and density) need to be chosen to derive the proper form of mass and energy balance equations

Given two independent properties, equations of state can be used to solve for the pressure variation with crank angle

- All other desired properties such as enthalpy, entropy, viscosity, etc. can also be calculated

Note: using proper units is essential when solving for pressure and temperature variations

- Use SI units
- Temperature [K], Pressure [Pa], Mass [kg], Time [s], Energy [J], Length [m]

Governing Equations (1/3)

RAY W. HERRICK LABORATORIES

- General form of mass balance:

$$\frac{dm_{cv}}{dt} = \frac{d(\rho V)}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

- Expanded form:

$$V\frac{d\rho}{d\theta} + \rho\frac{dV}{d\theta} = \left(\sum \dot{m}_{in} - \sum \dot{m}_{out}\right)\frac{1}{\omega}$$

(properties)

(geometry)

(leakage)

$\dot{m}_{in}$ = Mass flow rate into control volume, kg/s

$\dot{m}_{out}$ = Mass flow rate out of control volume, kg/s

P = Pressure, Pa

t = Time, s

T = Temperature, K

V = Volume of chamber, m^3

θ = Crankshaft angle, degrees

ρ = Density, kg/m^3

ω = Rotational speed of crank, deg/s

Governing Equations (2/3)

RAY W. HERRICK LABORATORIES

- General form of energy balance:

$$\frac{dE_{cv}}{dt} = \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} + \dot{Q} - \dot{W}$$

(properties) (leakage)

(geometry) (heat transfer)

- Assuming: E = U + ~~KE~~ + ~~KP~~ = mu

$$\frac{d(mu)}{d\theta} = \frac{1}{\omega}\left(\sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} + \dot{Q}\right) - P\frac{dV}{d\theta}$$

h = Specific enthalpy, J/kg

$\dot{m}_{in}$ = Mass flow rate into CV, kg/s

$\dot{m}_{out}$ = Mass flow rate out of CV, kg/s

P = Pressure, Pa

$\dot{Q}$ = Heat transfer rate into CV, W

T = Temperature, K

u = Specific internal energy, J/kg

V = Volume of chamber, m^3

W = Work done by control volume, W

θ = Crankshaft angle, degrees

ω = Rotational speed of crank, deg/s

ρ = Density, kg/m^3

Governing Equations (3/3)

- Expansion of internal energy (ρ, T) :

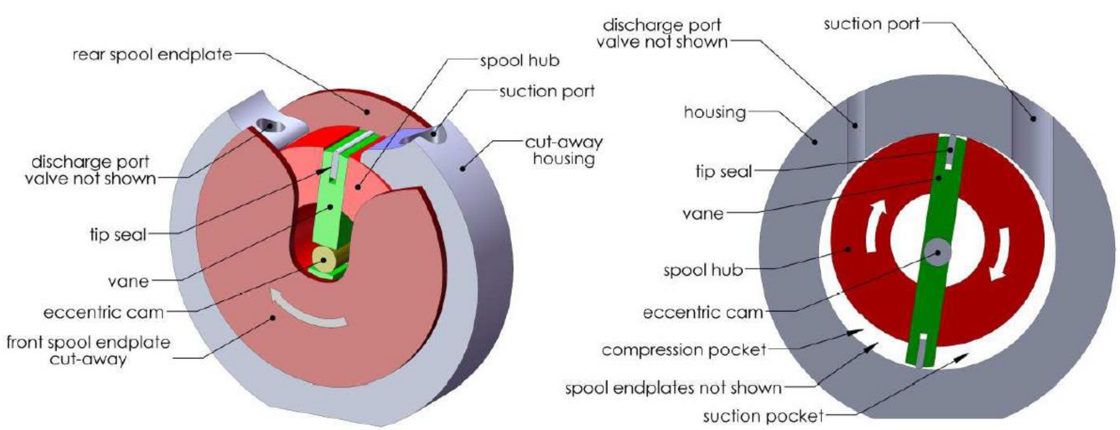
$$\frac{dT}{d\theta} = \frac{-\rho h \frac{dV}{d\theta} - \left(uV + \rho V \frac{\partial u}{\partial \rho} \right) \frac{\partial \rho}{\partial \theta} + \frac{1}{\omega} \left(\dot{Q} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out} \right)}{\rho V \frac{\partial u}{\partial T}}$$

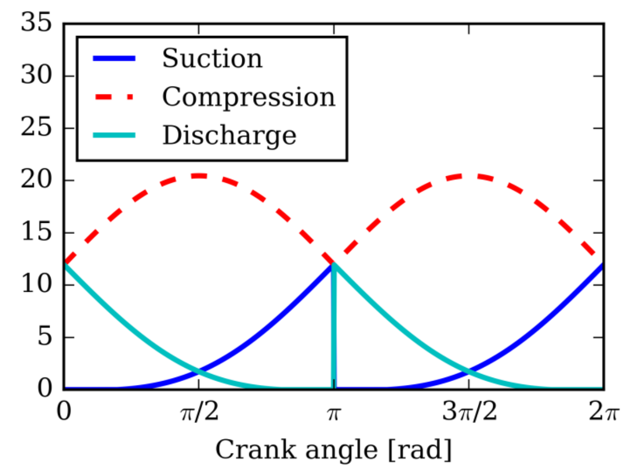
- Expansion of internal energy (v, T) :

$$\frac{dT}{d\theta} = \frac{-T \left(\frac{\partial p}{\partial T} \right)_v \left[\frac{dV}{d\theta} - v \frac{dm}{d\theta} \right] - h \frac{dm}{d\theta} + \frac{1}{\omega} \left(\dot{Q} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{in} h_{in} \right)}{m c_v}$$

Geometry Models (1/3)

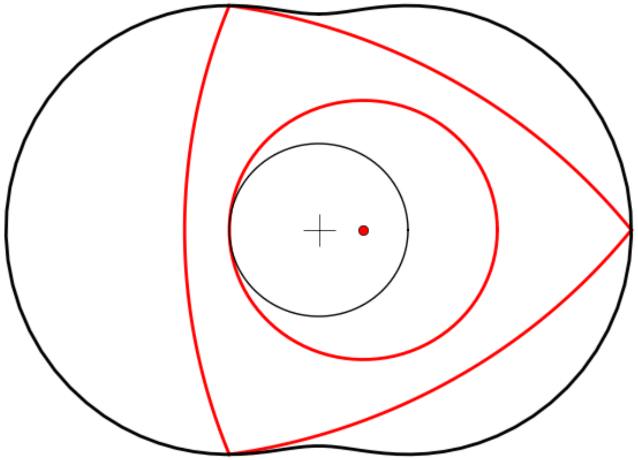
- **Calculate volume and volume derivative of working chambers**
- **For some geometries, analytic or quasi-analytic solutions exist**
  - For example: reciprocating, rolling piston, Z-compressor, rotary vane
- **Example of a spool compressor**





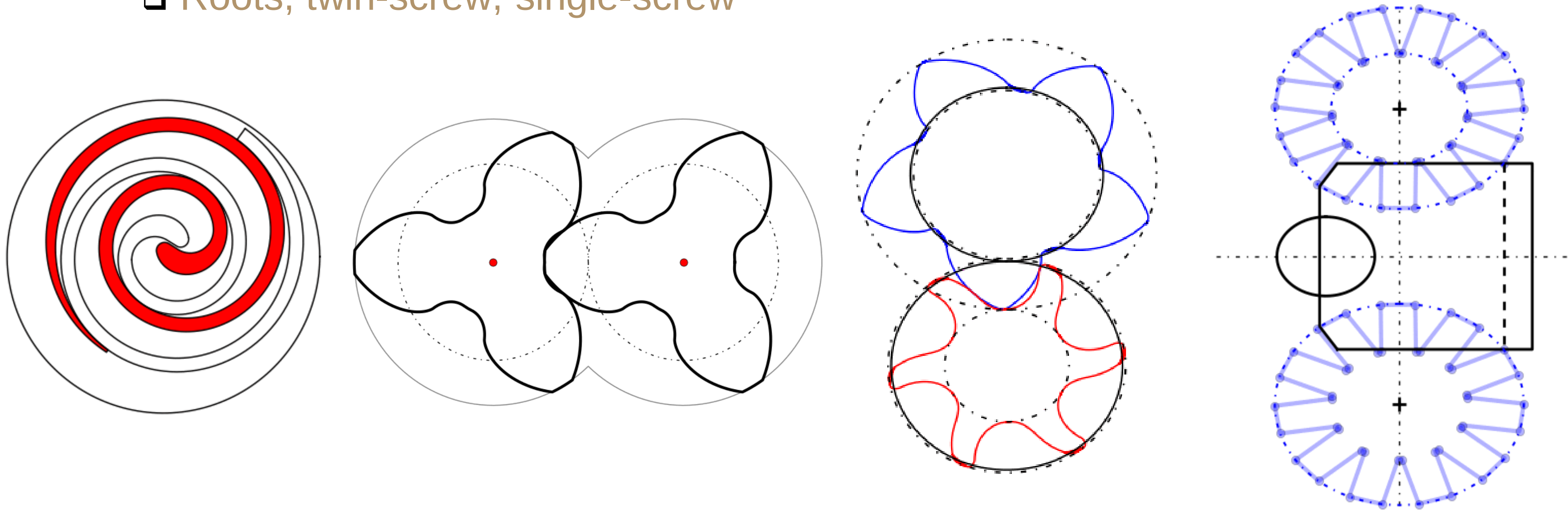
Geometry Models (2/3)

- **Calculate volume and volume derivative of working chambers**
- **For some geometries, analytic or quasi-analytic solutions exist**
  - For example: reciprocating, rolling piston, Z-compressor, rotary vane
- **Example of a Wankel compressor**



Geometry Models (3/3)

- **Other compressor types require more involved geometry modeling**
- **Polygon approach to describe curves and use Boolean operation**
  - ❑ Scroll
  - ❑ Roots, twin-screw, single-screw



Flow Models (1/5)

Fluid is moved through a compressor mainly by pressure difference

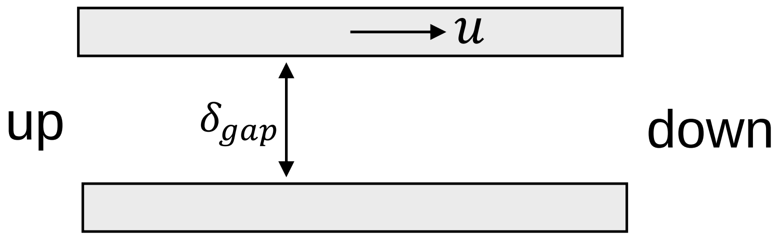
Flow through long path relatively to the gap size might be affected by friction

Mass flow models will determine how fluid moves through the different flow paths in the compressor

The functional formulation of a general mass flow model is given as

$$\dot{m} = f(T_{up}, p_{up}, p_{down}, C_{flow}, A_{path}(\delta_{gap}, D_h))$$

Typical flows: valves, leakages



Flow Models (2/5)

Model this as flow through a nozzle (Fox et al., 2004)

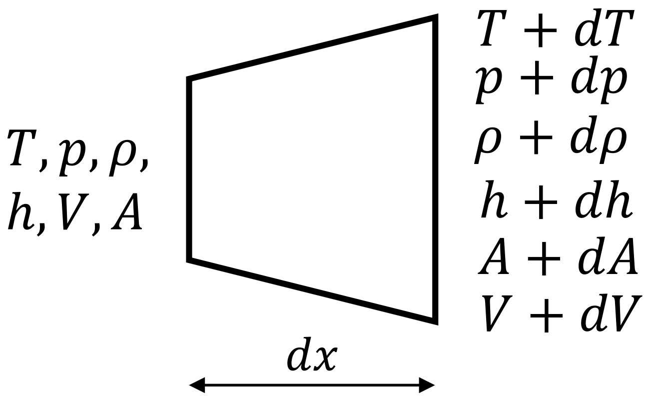
- Compressible, adiabatic (no heat transfer)
- Frictionless
- Isentropic (constant entropy)

Can add more complicated flows if needed

- Fanno/Rayleigh flows (Fox et al., 2004)
- Empirical mass flow relations (Ishii et al., 2008)
- Couette-Poiseuille flow

Flow with Friction

- Control volume analysis applied to leakage flow
  - No heat transfer, no mass transfer
  - Variable area, 1-D description
  - Compressible flow
  - Real gas properties
  - No oil



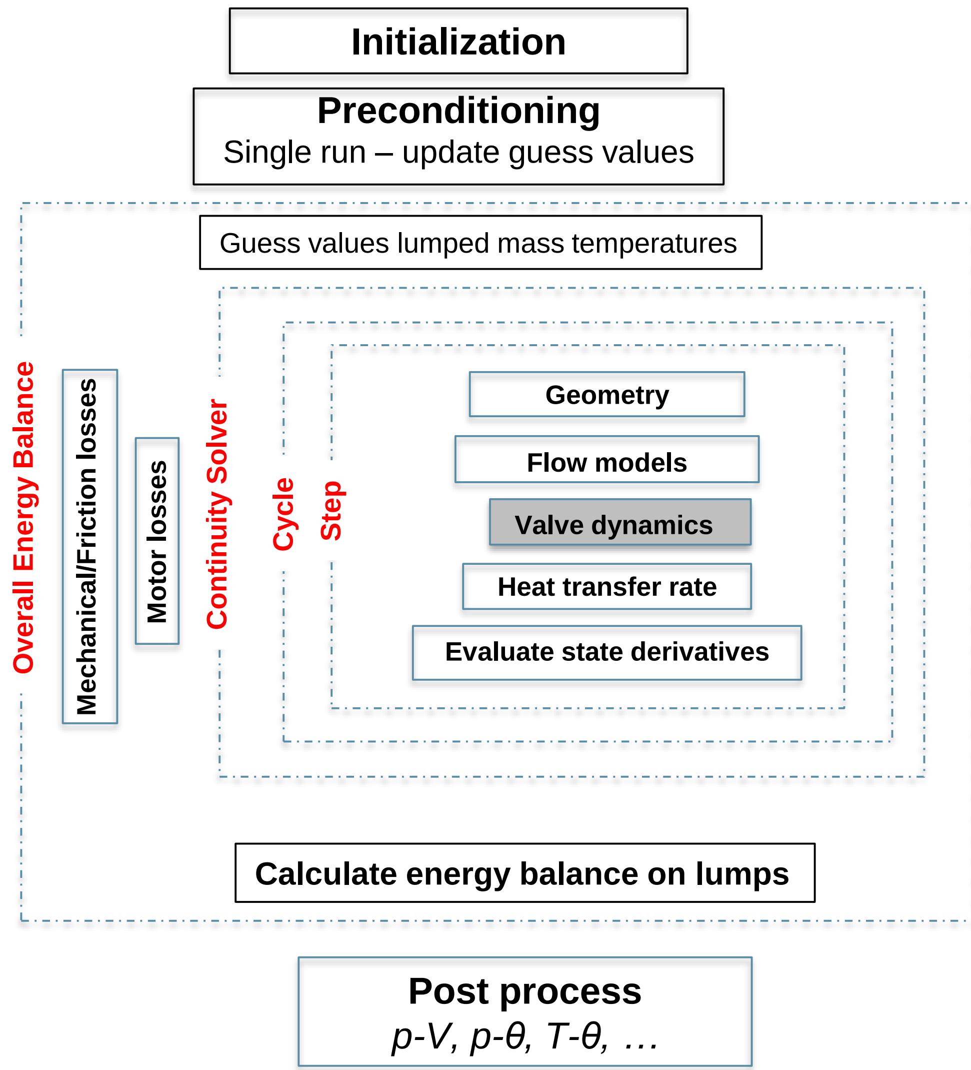
- Apply continuity, momentum and energy balance equations

Flow Models (3/5)

- The valve model is a sub-step in the mass-flow calculations
- The mass flow-rate through a valve is dependent on the area of the valve openings and the flow conditions

$$\dot{m} = f\big(\underbrace{p_{high}, p_{low}}_{\text{From current state point}}, \underbrace{A(x_{valve})}_{\text{From valve model}}\big)$$

- Common valves in compressors are:
  - ✓ Reed valves
  - ✓ Poppet valves or plug valves
  - ✓ Flat-plate spring backed valves



Flow Models (4/5)

- The equation of motion applied to a free-body is:

$$m_v \ddot{x}_v(t) + C_v \dot{x}_v(t) + k_v x_v(t) = F(t)$$

  - For poppet valves: the valve stiffness $k_v = const$
  - For reed valves: the valve stiffness $k_v = k_v(x_v)$

- The equation of motion can also be written as:

$$\ddot{x}_v(t) + 2\xi\omega_n \dot{x}_v(t) + \omega_n^2 x_v(t) = \frac{F(t)}{m_v}$$

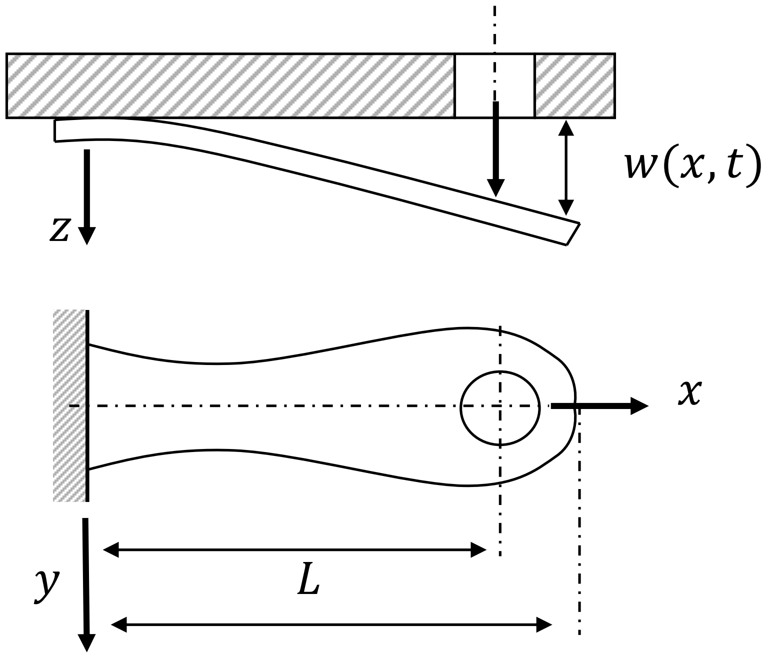
where:

- ω_n [rad/s]: Natural frequency $\omega_n = \sqrt{\frac{k_v}{m_v}}$
- ξ damping coefficient
- ξ_{crit} critical damping constant $\xi = \frac{C_v}{\xi_{crit}} = \frac{C}{2 m_v \omega_n}$

Flow Models (5/5)

- Valve dynamics characterized by the vibration of the valve plate
- Valve plate is described as a beam with varying width
- Shear and rotational motion are neglected
- The valve deflection is given as:

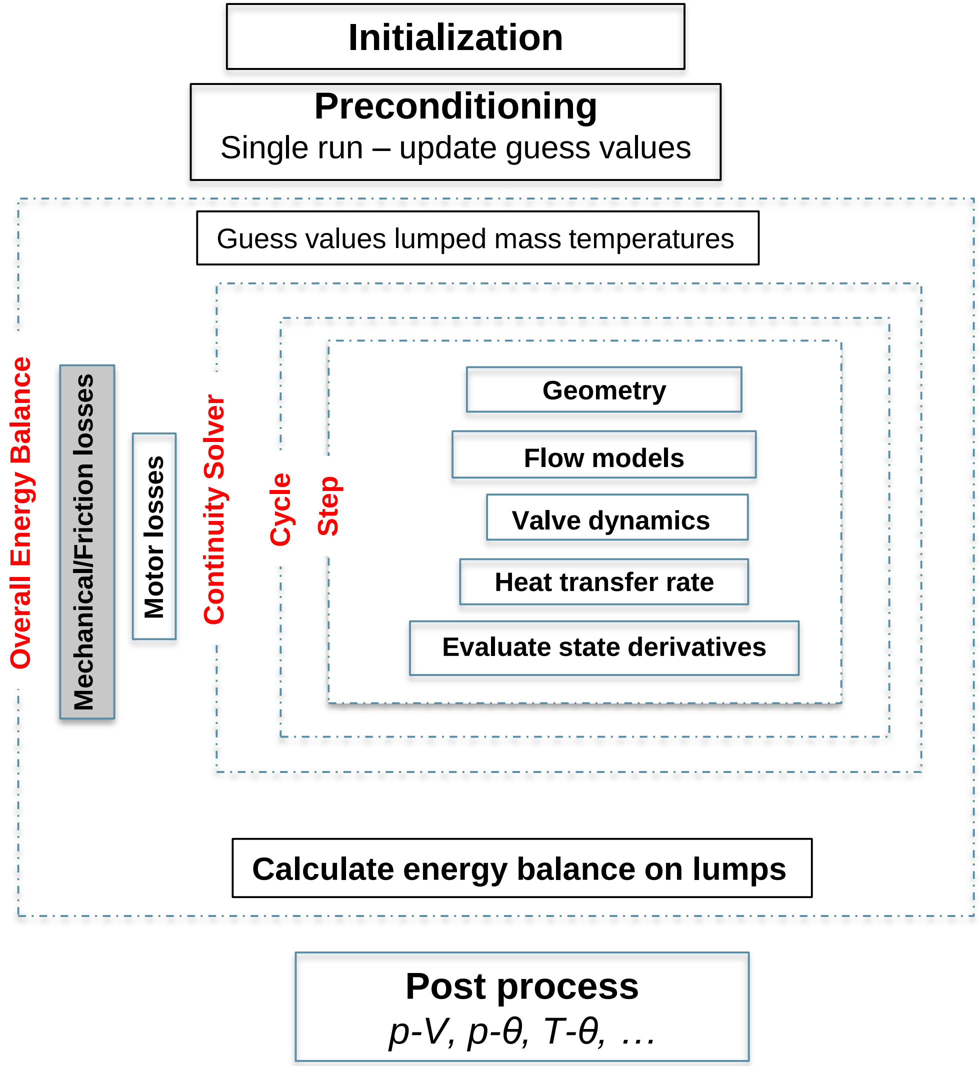
$$y = \sum_{n=1}^{m} \phi_n(x) q_n(t)$$



$\phi_n(x)$ valve shape function (can be determined from free vibration analysis). There exists infinite number of combinations of mode shapes

$q_n(x)$ generalized coordinate or mode participation factor which may be obtained by integrating the valve governing equation

Friction Losses (1/3)



Friction Losses (2/3)

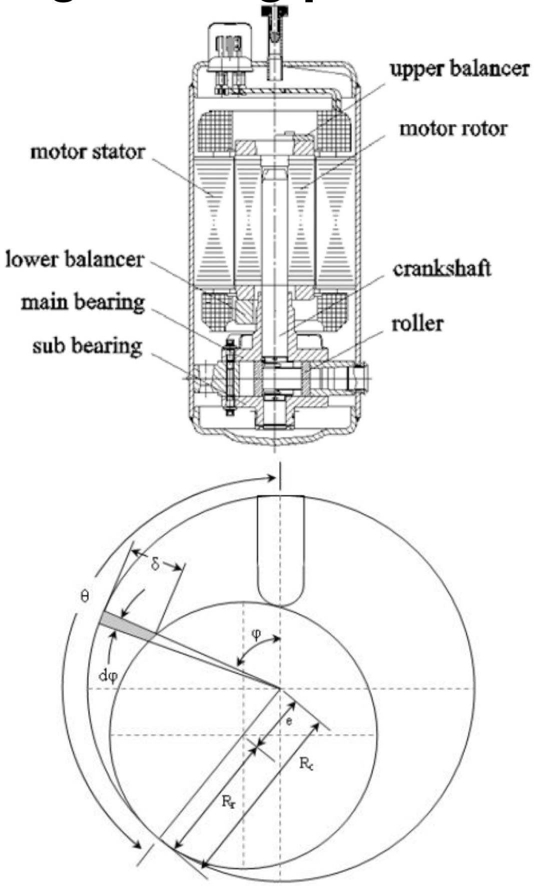
- **Numerous models available with varying level of empiricism:**

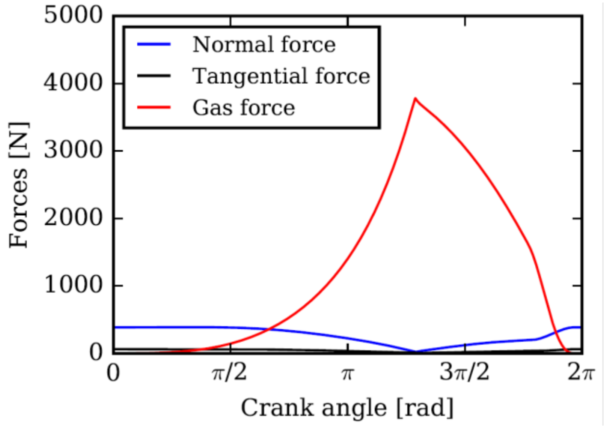
Mechanistic → **Empirical**

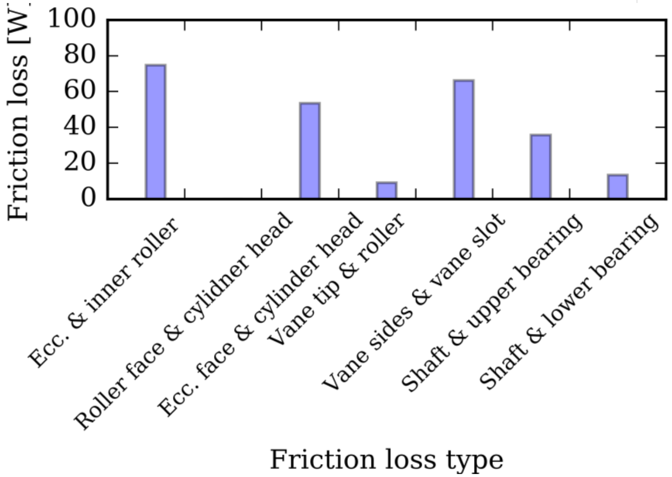
Detailed dynamics and frictional model	Correlation of semi-empirical parameters	Correlation of mechanical losses
	$\dot{W}_{ML} = f\left(\tau, \dot{W}_{gas}, \ldots\right)$	$\dot{W}_{ML} = f\left(T_s, T_d, \ldots\right)$

Friction Losses (3/3)

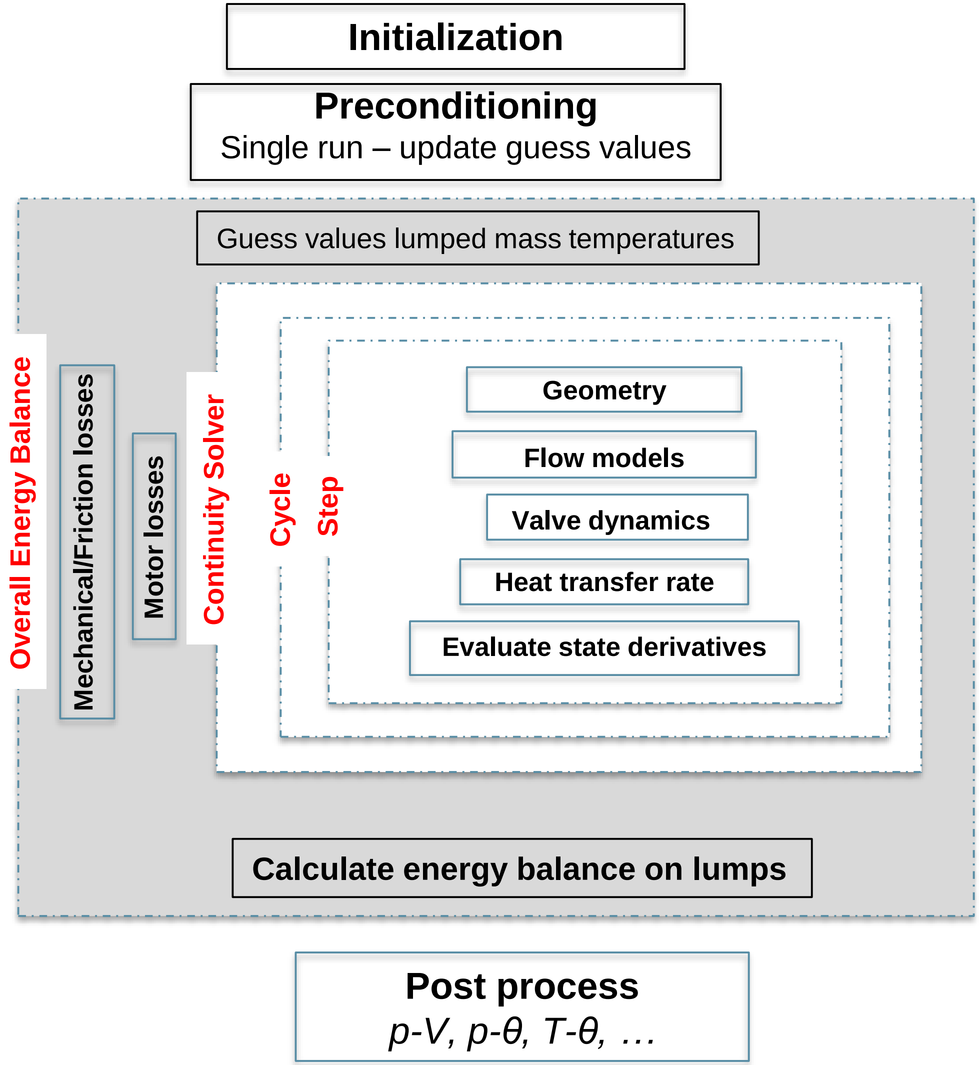
- Example of force analysis and friction losses on a single-stage rolling piston compressor:





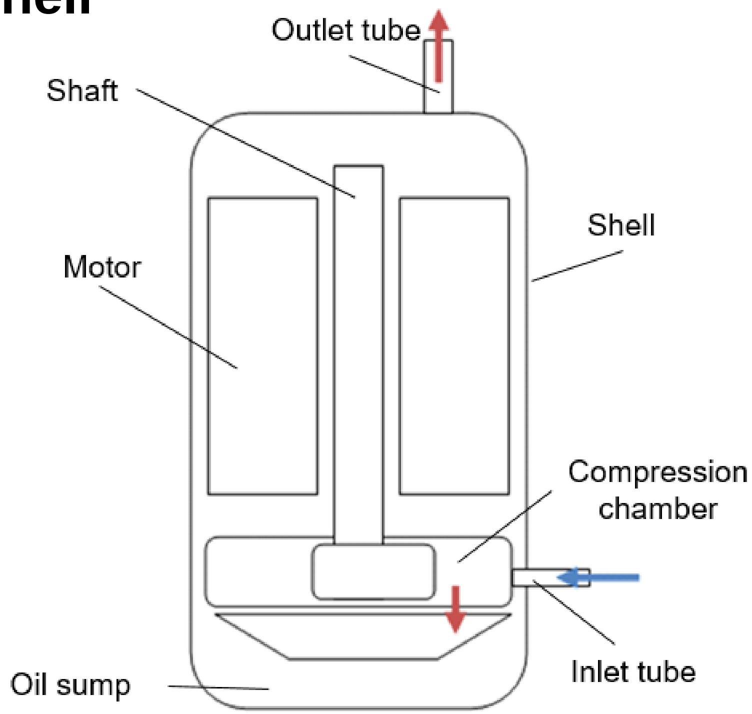


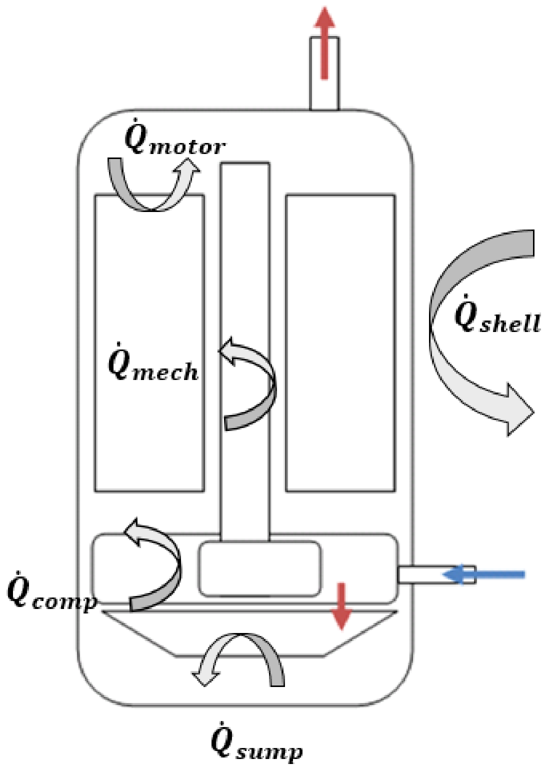
Overall Energy Balance (1/4)



Overall Energy Balance (2/4)

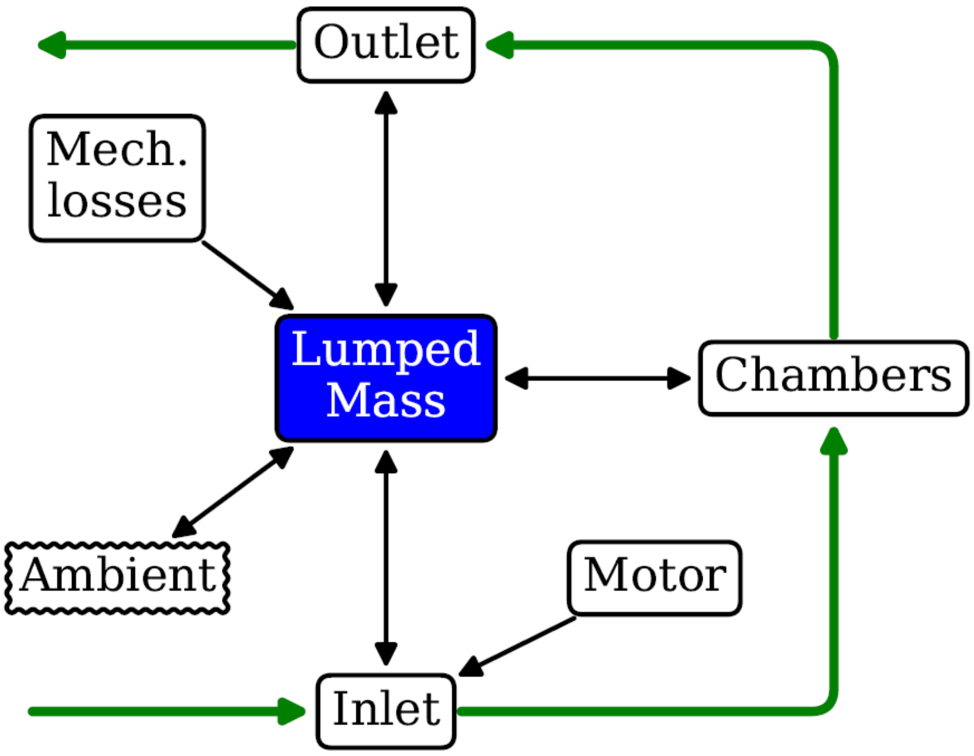
- A general compressor structure includes several elements
- Complex thermal interactions occurs between each element
- It is important to quantify the heat losses through compressor shell





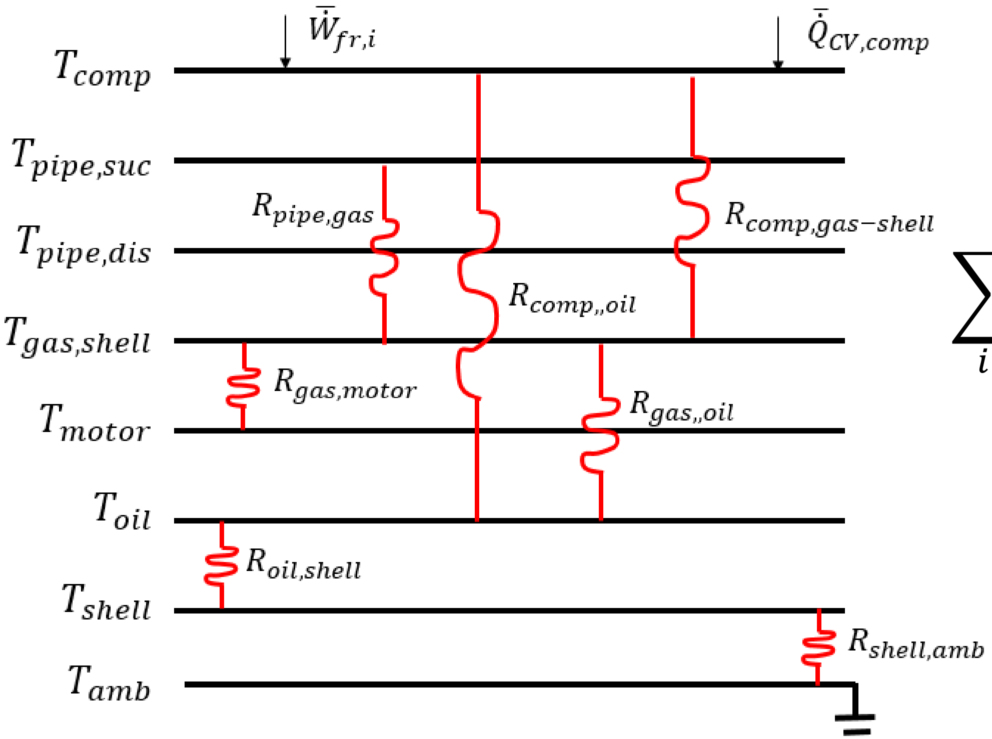
Overall Energy Balance (3/4)

- Single-lumped temperature



Overall Energy Balance (4/4)

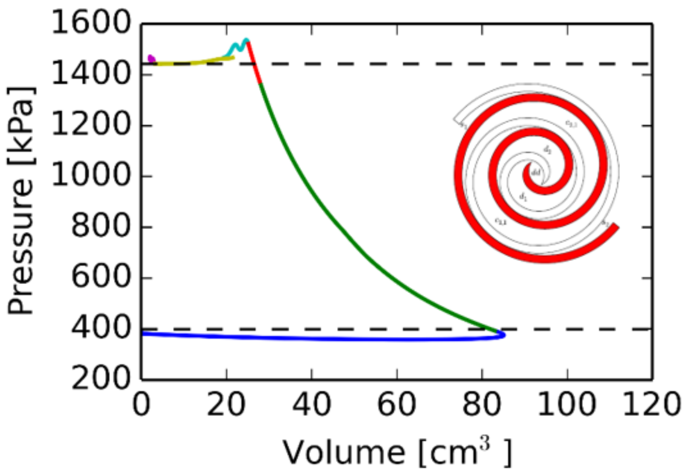
- **Multi-lumped temperatures**

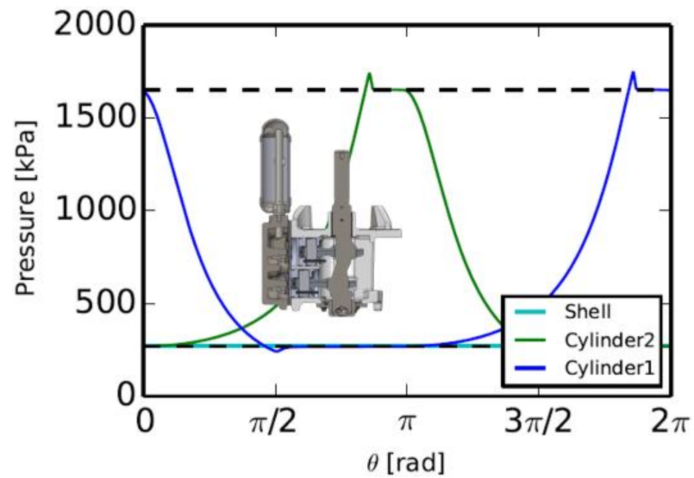


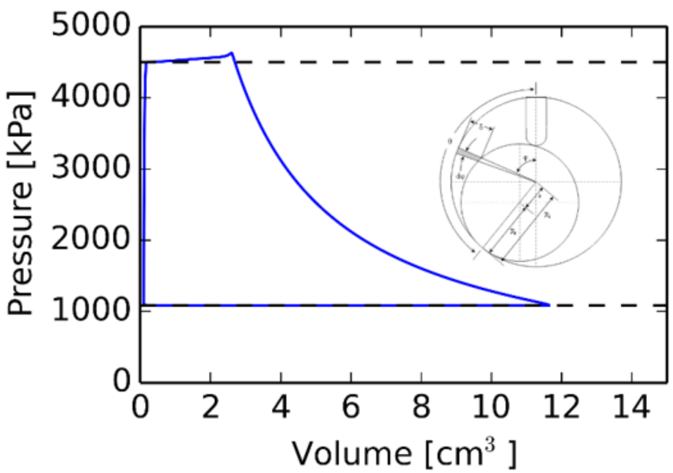
$$\sum_i \left(\dot{Q}_{lump,in,i} - \dot{Q}_{lump,out,i}\right) + \dot{Q}_{lump,gen} = 0$$

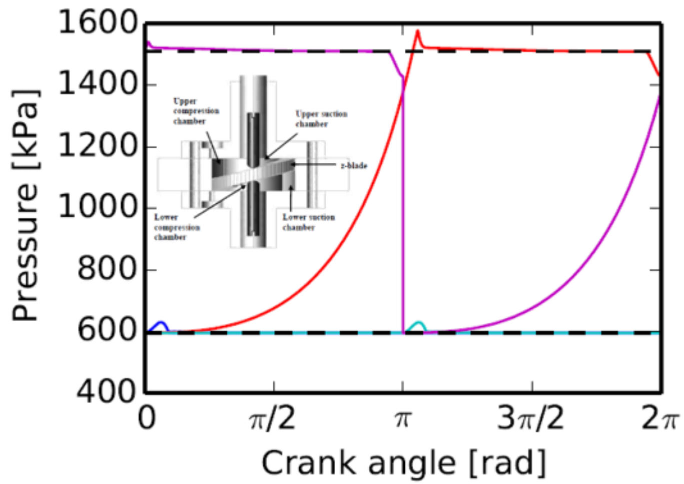
$$\dot{Q}_{\text{lump}} = \frac{T_{\text{lump,a}} - T_{\text{lump,b}}}{R_{\text{ab}}}$$

Example of Compressor Models





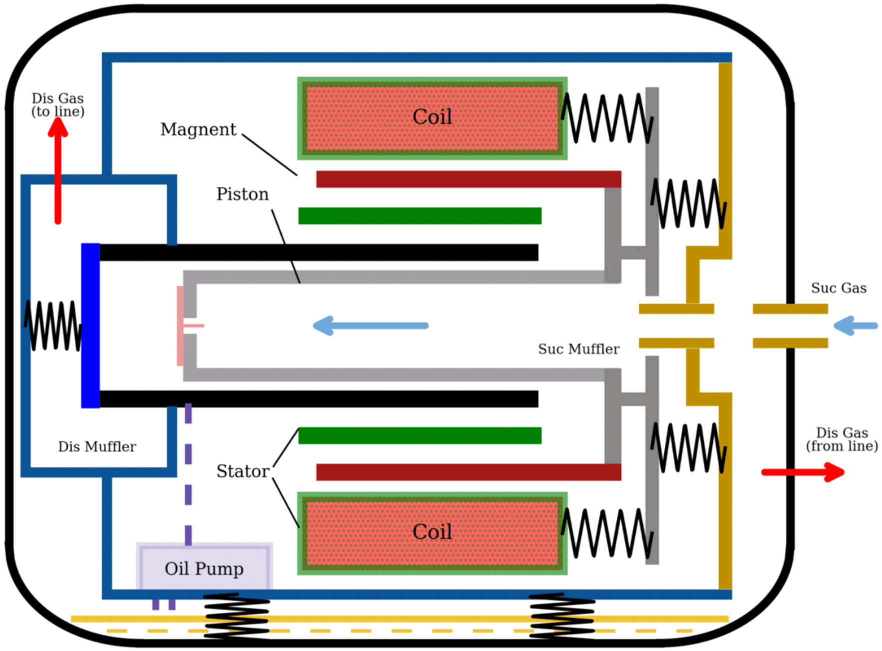




Outline

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

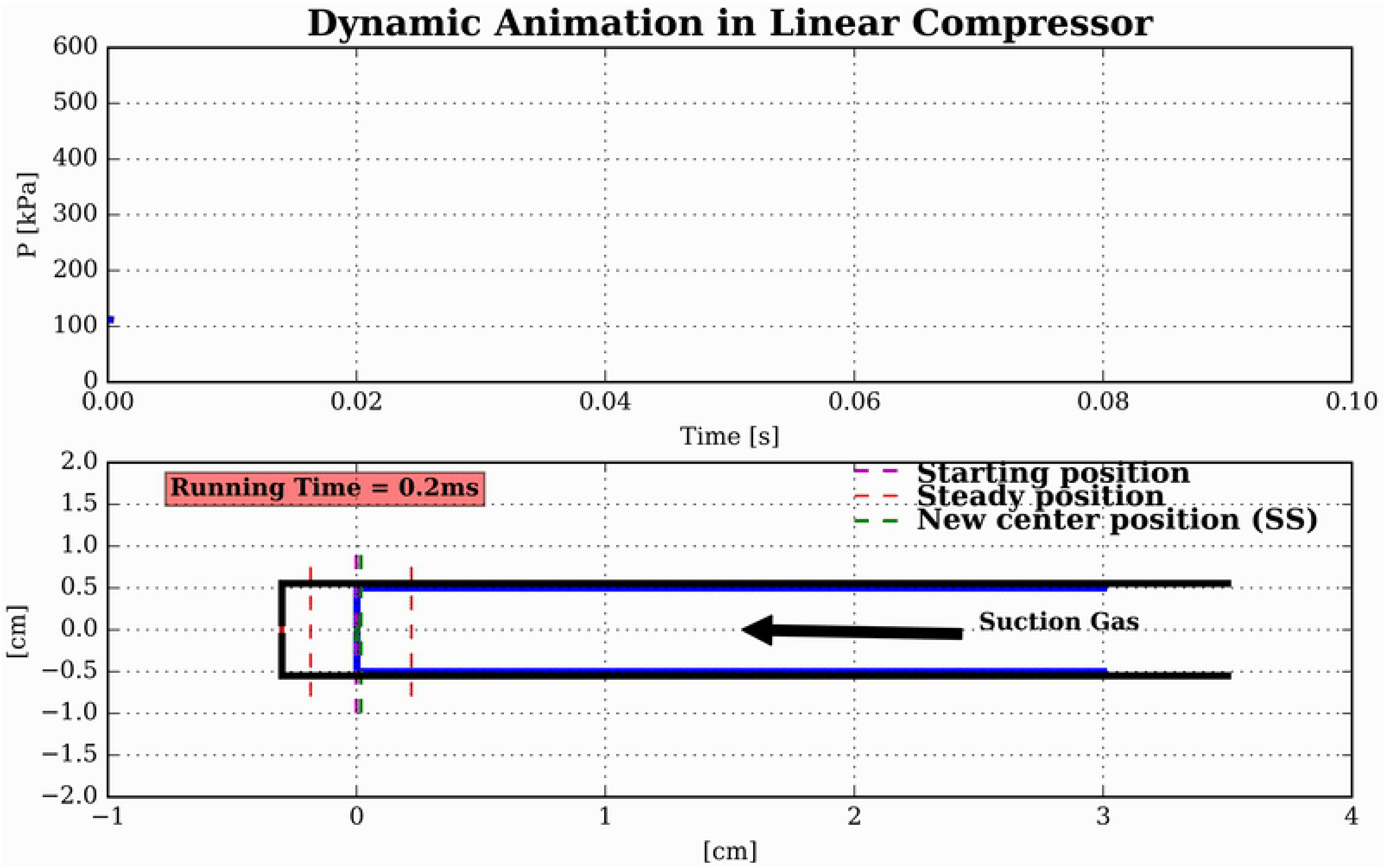
Linear Compressor (1/6)



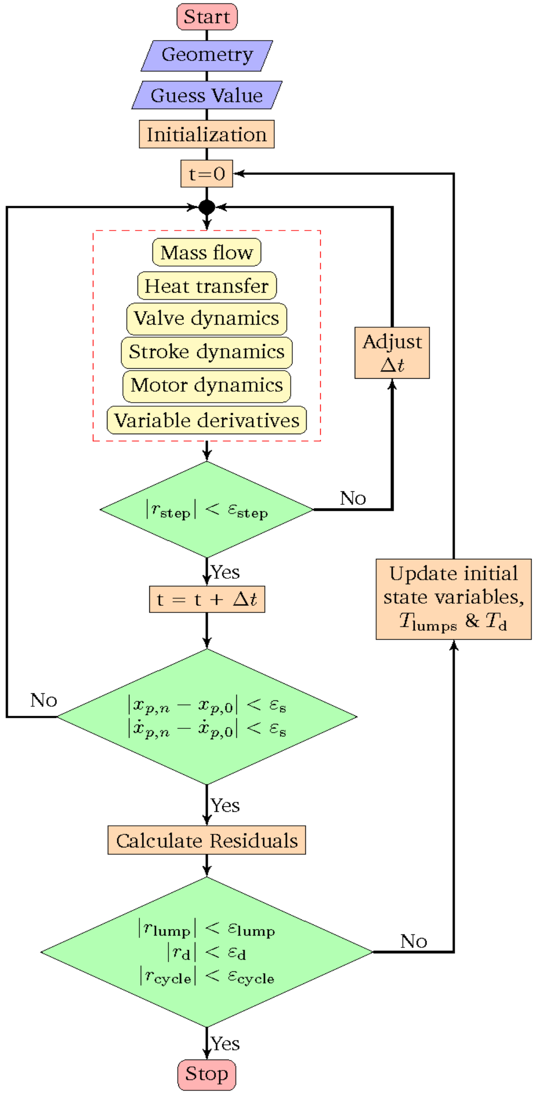
Characteristics of linear compressor

- Piston is driven by moving magnet oscillating motor
- Stroke is not fixed by a crank mechanism but is instead determined by design and operating conditions
- Operate at resonance frequency to get the higher motor efficiency
- All comprehensive compressor model is time dependent not rotation (cracking angle) dependent.

Linear Compressor (2/6)



Linear Compressor (3/6)



System Geometry

- Piston geometry
- Frequency
- Valve geometry

Given Condition

- Inlet temperature/pressure
- Pressure ratio

Guess value

- Mass flow rate
- Lumped temperatures
- Piston/valve initial conditions

Linear Compressor (4/6)

RAY W. HERRICK LABORATORIES

Governing equations:

$$m_{cv}C_{cv}\frac{dT}{dt}+T\left(\frac{\partial P}{\partial T}\right)_v\left[\frac{dV}{dt}-\frac{1}{\rho}\frac{dm_{cv}}{dt}\right]+h_{cv}\frac{dm_{cv}}{dt}=\dot{Q}+\sum\dot{m}h_{in}-\sum\dot{m}h_{cv}\rightarrow \text{Energy Balance}$$

$$\frac{dm_{cv}}{dt}=\frac{dm_{in}}{dt}+\frac{dm_{leak,in}}{dt}-\frac{dm_{out}}{dt}-\frac{dm_{leakage,out}}{dt}\rightarrow \text{Mass Balance}$$

$$m\ddot{x}_p+c_{fri}\dot{x}_p+k_s x_p+(P(t)-P_{shell})A_p=\alpha I(t)\rightarrow \text{Piston Motion}$$

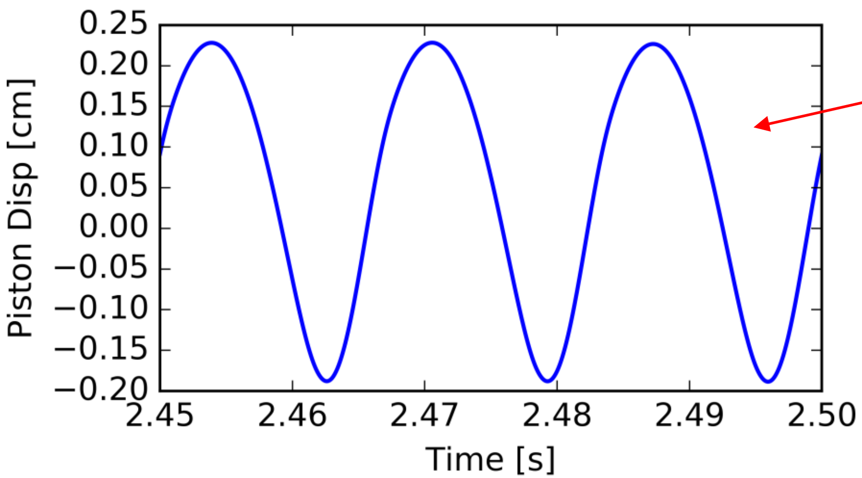
$$m_{eff}\ddot{y}_v+\frac{1}{2}C_D\rho A_v V_{gas}{}^2(t)A_v+k_v y_v=F_v(t)\rightarrow \text{Valve Motion}$$

$$V_e-\alpha\dot{x}_p=L\dot{I}+RI+\frac{1}{C}\int I dt\rightarrow \text{Motor Dynamics}$$

Sub-models:

- Heat transfer model
- Friction model
- Overall Energy Balance

Linear Compressor (5/6)

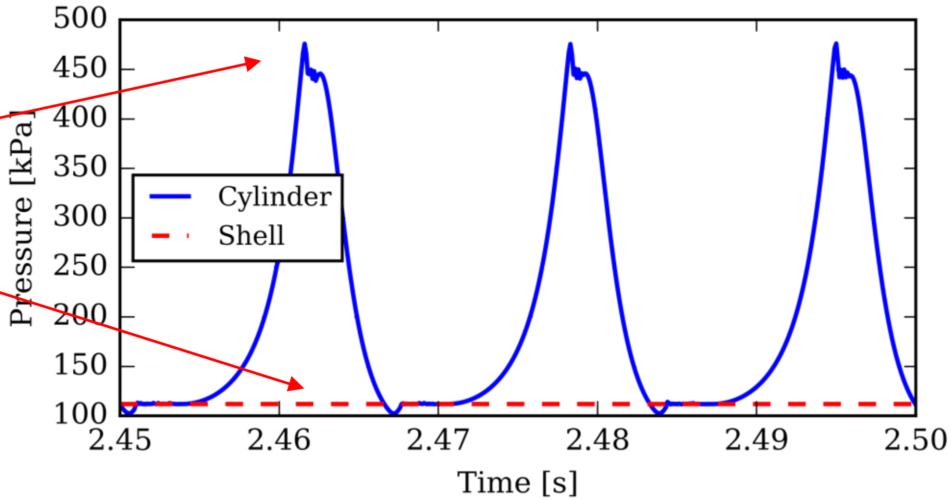


Piston Stroke is approximately 0.4 cm

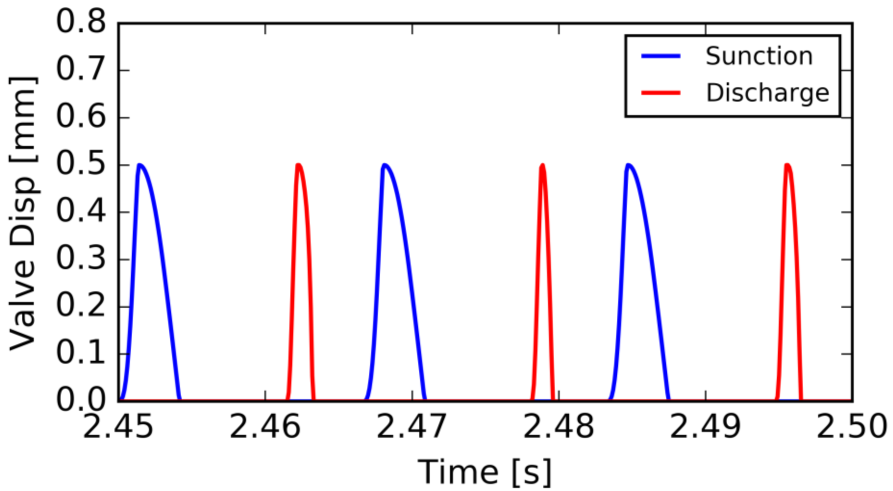
Two different type valves:

Plate valve shows vibration

Reed valve opens smoothly

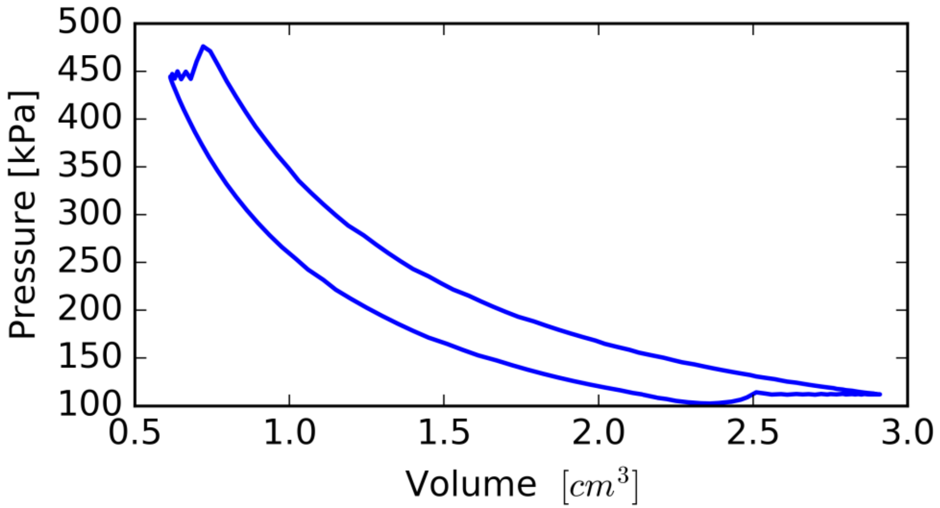


Linear Compressor (6/6)



- Valve stopper was set to 0.5 mm
- Suction valve has longer opening time than discharge valve
- Two valves show different performance

- Linear compressor has larger clearance volume
- Discharging process shows obvious vibration due to the use of plate valve

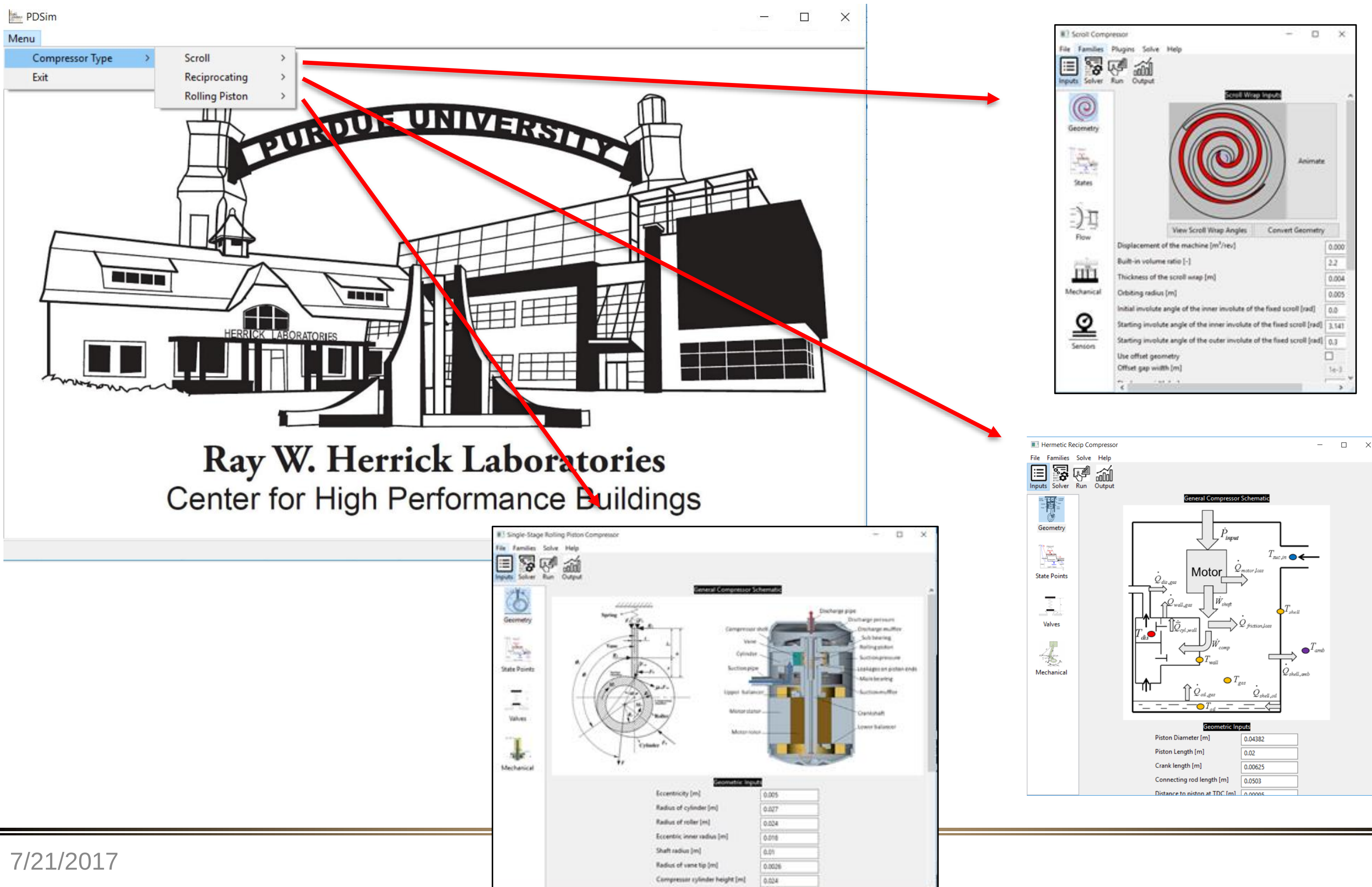


Outline

RAY W. HERRICK LABORATORIES

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Graphical User Interface (1/2)



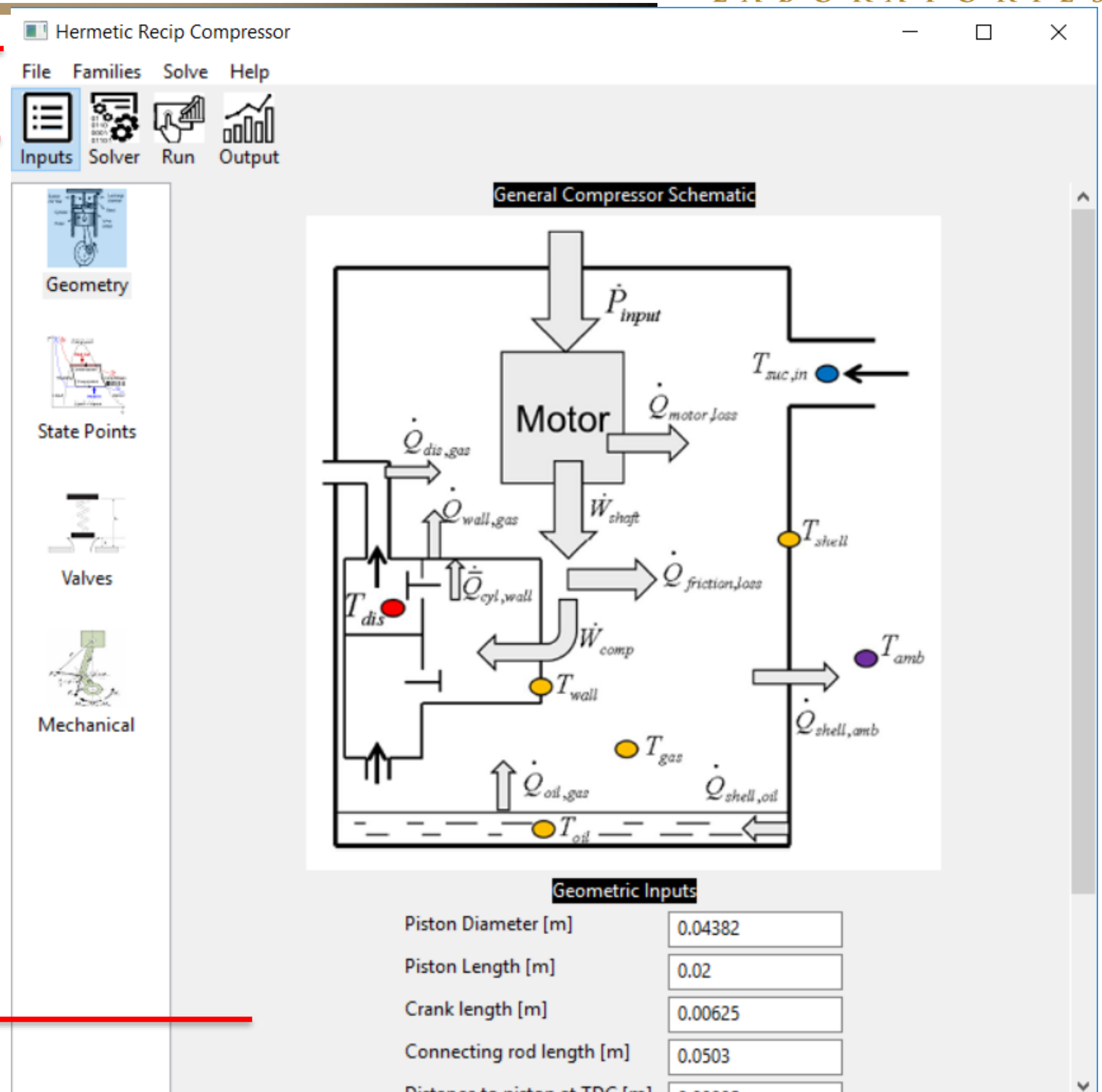
Graphical User Interface (2/2)

Compressor type

Model setup

Sub-models

User inputs



Outline

- Introduction
- History of positive displacement modeling
- Generalized modeling structure (PDSim)
- Control volume analysis
- Steady-periodic modeling (crank-angle based)
- Dynamic modeling (frequency-driven)
- Graphical User Interface (GUI)
- Conclusions

Conclusions

- Compressor modeling and testing have been carried out at Ray W. Herrick Laboratories for more than 40 years
- Mechanistic models have been proven to be extremely useful in accurately predicting compressor performance and identifying improved compressor designs
- A generalized platform has been developed to cover the majority of positive displacement machines
- The highly object-oriented core structure enhances the capabilities of the tool to be adapted to new compressors types

Acknowledgments

RAY W. HERRICK LABORATORIES

Center for
High Performance
Buildings at Purdue

A Center Dedicated to Partnering with Industry in the Development, Demonstration, Evaluation, and Deployment of new Technologies and Analysis Tools for High Performance Buildings

https://engineering.purdue.edu/CHPB

2018 Purdue Conferences

RAY W. HERRICK LABORATORIES

Questions

RAY W. HERRICK LABORATORIES

Eckhard A. Groll
Reilly Professor of Mechanical Engineering
E-mail: groll@purdue.edu

Davide Ziviani
Research Associate
E-mail: dziviani@purdue.edu

Purdue University, Ray W. Herrick Laboratories
West Lafayette, Indiana 47907, USA

References

- Bell, I.H., Wronski, J., Quoilin, S., Lemort, V., 2014. Pure and pseudo-pure fluid thermophysical property evaluation and the open-source thermophysical library CoolProp. Industrial & Engineering Chemistry Research 53 (6), 2498-2508.
- Bradshaw, C., 2012. A miniature-scale linear compressor for electronics cooling. Ph.D. thesis, Purdue University.
- Jovane, M., 2007. Modeling and analysis of a novel rotary compressor. Ph.D. thesis, Purdue University
- Hamilton, J. F., 1974. Extension of Mathematical Modeling of Positive Displacement Type Compressors. Short Course Text in Ray Herrick Laboratories, School of Mechanical Engineering, Purdue University.
- Kim, J.-H., 2005. Analysis of a bowtie compressor with novel capacity modulation. Ph.D. thesis, Purdue University
- Lemmon, E.W., Huber, M.L., McLinden, M.O., 2013. NIST Standanrd Reference Database 23: Reference Fluid Thermodynamic and Transport Properties-REFPROP, Version 9.1.1
- Liu, Z., 1994. Simulation of a Variable Speed Compressor with Special Attention to Supercharging Eects. Ph.D. thesis, Purdue University.

References

- Soedel, W., 1972. Introduction to Computer Simulations of Positive Displacement Type Compressors. Short Course Text in Ray Herrick Laboratories, School of Mechanical Engineering, Purdue University.
- Yang, B., 2017. Ph.D. thesis, Purdue University.
- Ziviani, D., 2017. Theoretical and Experimental Characterization of Single-Screw Expanders for Organic Rankine Cycle Applications. Ph.D. thesis, Ghent University.